Kernelization of vertex cover based on crown structure

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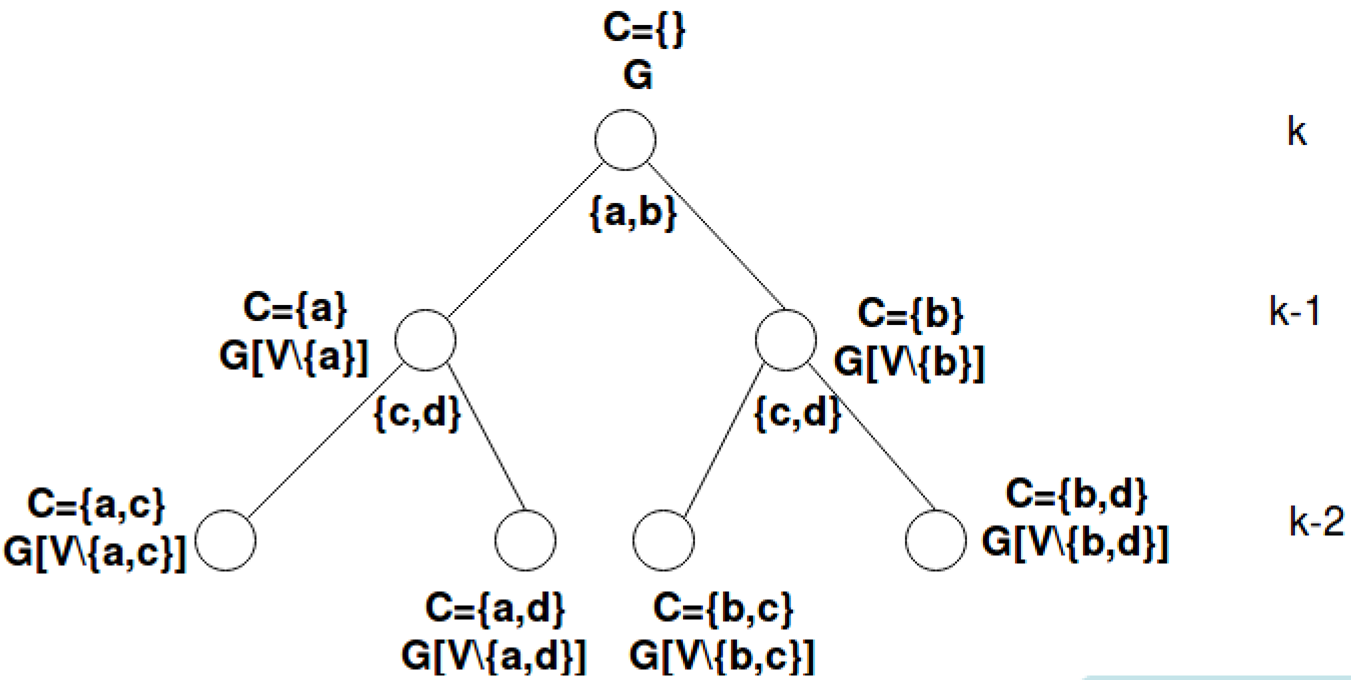
Vertex Cover problem

Given a graph G=(V,E), find a subset C of V of size at most k such that for each edge {u,v} in E, either u or v belongs to C



Time complexity of this solution is O((n+m)2^k).

Fixed parameter tractable algorithms

- Instead of expressing the running time as a function T(n) of n, we express it as a function T(n,k) of the input size n and some parameter k of the input

- Algorithms whose running times are exponential only in a chosen parameter, and are polynomial in the instance size

- *T(n,k)=O(poly(n)f(k))* where f() is an exponential function and poly() is a polynomial function

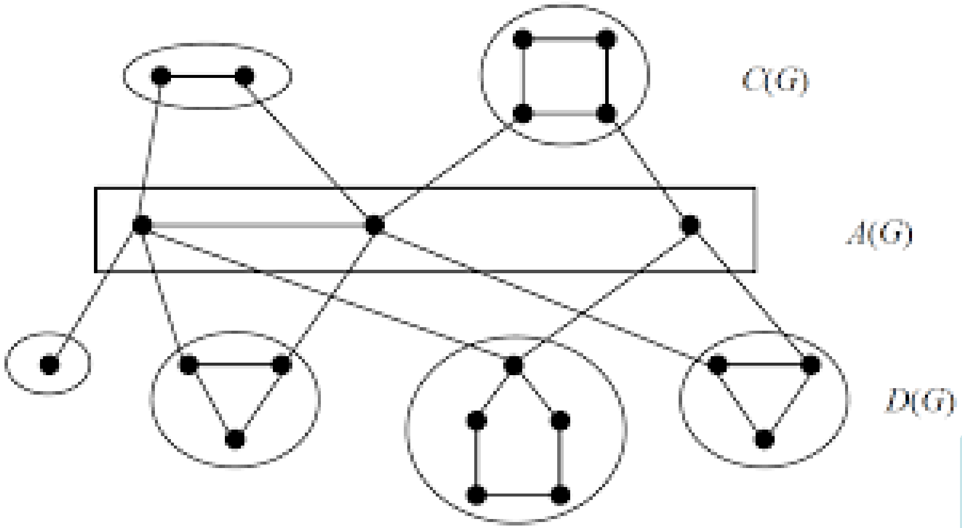
- Efficient on all inputs where k is small.

Kernelization

- Reduce problem instance (I,k) to (I',k')
- Size of I' is *poly(k)*
- Time to solve reduced instance is *poly(|I'|,k')*
- $k' <= k$
- Reduction takes polynomial time
- Reduced instance has a solution iff given instance has a solution

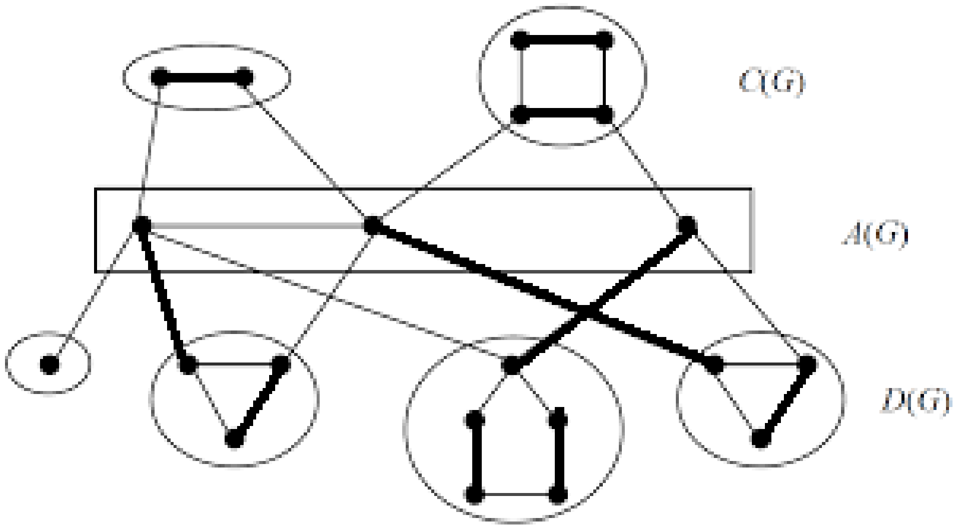
Kernelization method

- The goal is to reduce the problem instance for vertex cover problem
- The reduced instance will have at most 2k vertices
- Gallai-Edmonds decomposition



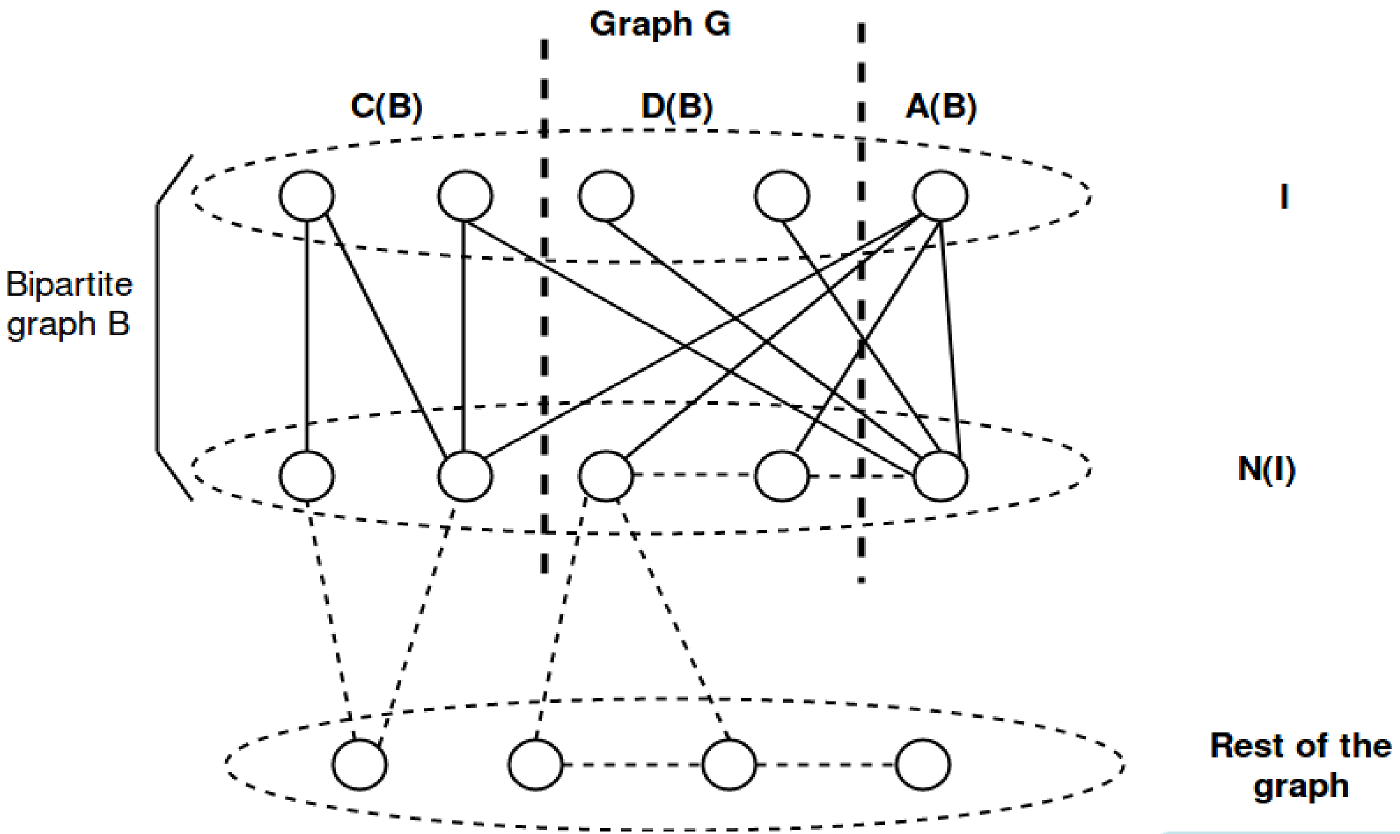
Source: Graphs of Edge-Intersecting and Non-Splitting Paths PART II: One-Bend ENPG

Properties of Gallai-Edmonds decomposition



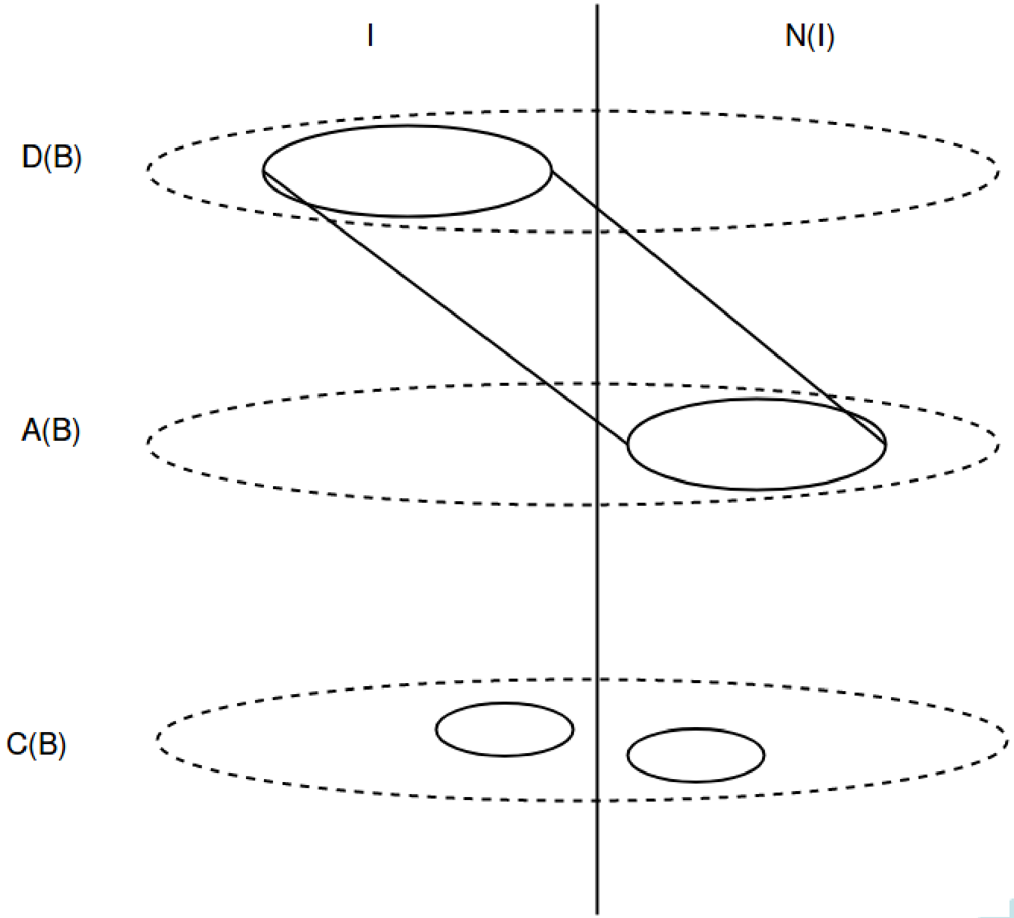
- C(G) and A(G) vertices are matched in all maximum matchings
- A(G) vertices are matched to D(G) vertices and C(G) is perfectly matched in a maximum matching
- D(G) components are near-perfectly matchable. One vertex is unmatched or matched outside.
- D(G) is an independent set in bipartite graphs

Reduction



Set of isolated vertices of D(G) and their neighbors denoted by I and N(I) respectively

Reduction



Bipartite graph B

There exists a minimum vertex cover of G which does not contain any vertex from (C(B)∪D(B))∩I and contains all vertices from (C(B)∪A(B))∩N(I).

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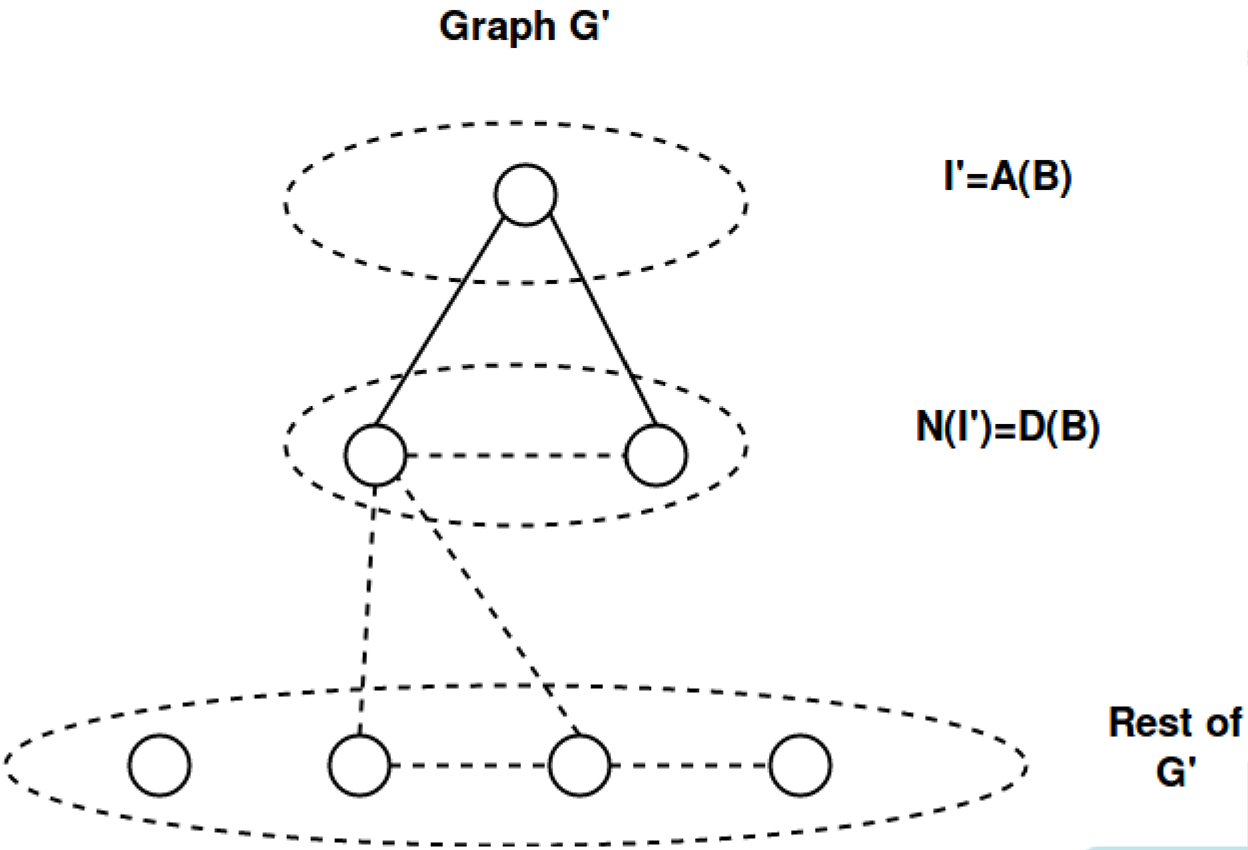
Reduction



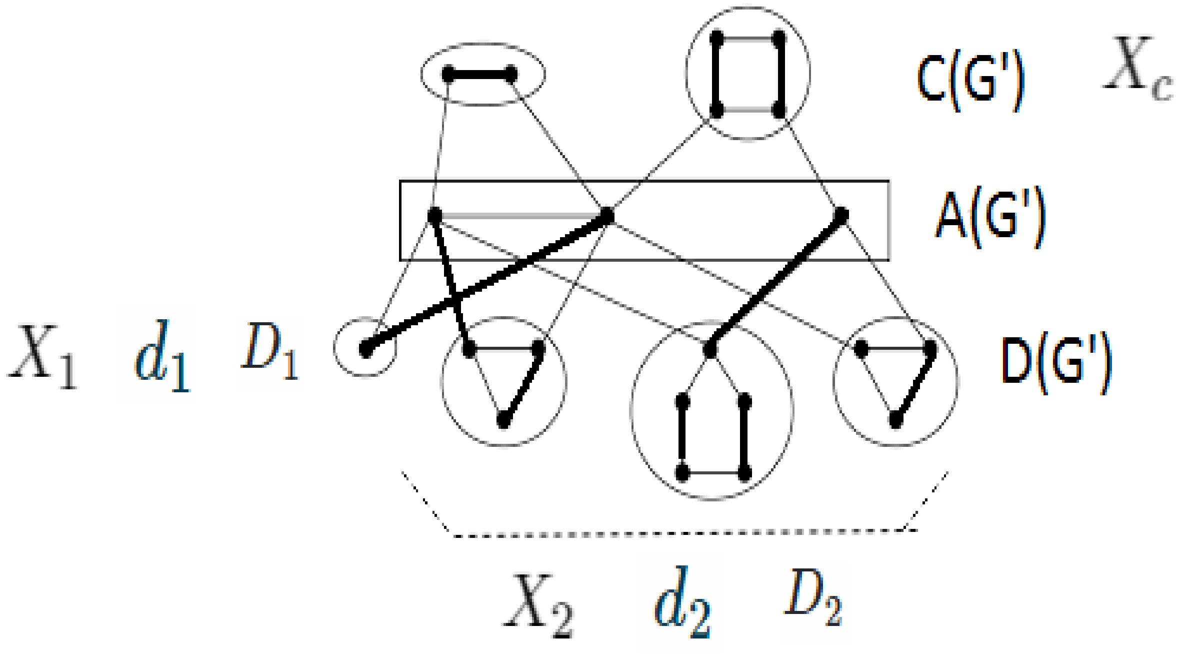
Reduced Graph G’

xperimental work)

If C′ is a minimum vertex cover of the reduced graph G′, then C = C′∪[(C(B)∪A(B))∩N(I)] is a minimum vertex cover of G.

There exists a maximum matching for G′ in which all I′ vertices are matched.

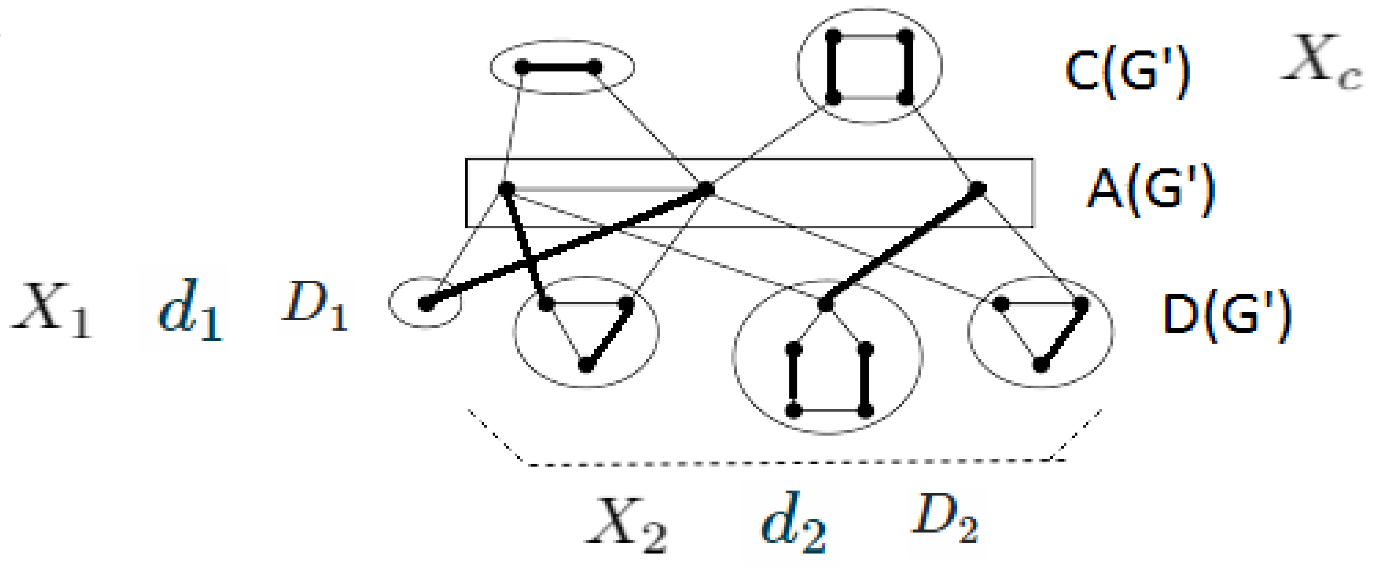
2k kernel



Gallai-Edmonds decomposition of G'

Let M be the maximum matching for G' in which all vertices of I' are matched.

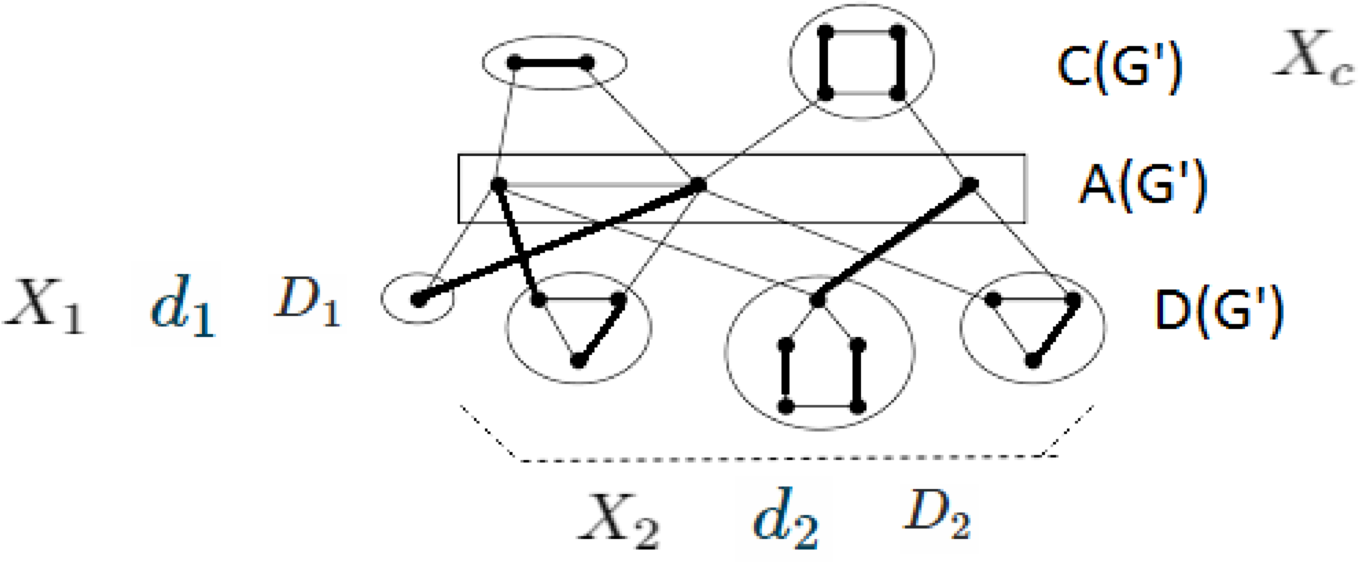
2k kernel



Gallai-Edmonds decomposition of G'

- $|A(G')| <= D_1+D_2$ from properties of Gallai-Edmonds decomposition
- $|A(G')|+|D(G')| <= D_1+D_2+d_1+d_2$
- $X_1 >= d_1$ because in M, all d_1 vertices are matched
- $X_2 >= (d_2 + D_2)/2$ because for any component of D(G') with size 2r+1 where r >= 1, at least r+1 vertices are required to cover the vertex cover

2k kernel



Gallai-Edmonds decomposition of G'

- $|A(G')|+|D(G')| <= D_1+D_2+d_1+d_2 = 2D_1 + 2(D_2+d_2)/2 <= 2(X_1+X_2)$
- $|C(G')| <= 2|X_C|$ because number of matching edges of C(G') is exactly $|C(G')|/2$
- $|V|=|D(G')|+|A(G')|+|C(G')| <= 2(|X_1|+|X_2|+|X_C|) <= 2|X|$

Conclusions

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- An algorithm was presented for reducing the problem instance for vertex cover to a graph of size 2k, where k is size of vertex cover
- Finding a minimum vertex cover find its applications in keyword based text summarization.
- Simulating propagation of computer worms on a network and designing techniques to prevent them also make use of minimum vertex cover

References

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